Elastic Constants for Multilayered Sandwich Cylinders

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Nomenclature

B_{rs},C_{rs},D_{rs} = elastic stiffness constants of shell [Eqs. (19–22)]
$E_x{}^i,E_y{}^i,E_{xy}{}^i$ = moduli of elasticity of the ith facing
$G_{x\theta}{}^i$ = shearing modulus of the ith facing
$G_{xz}{}^i,G_{z\theta}{}^i$ = shearing modulus of the ith core
h_i = thickness of the ith core
$K_{rs}{}^i$ = elastic constant of the ith facing or core [Eq. (10)]
m,n = number of facings and cores, respectively
M_x,M_y,M_{xy} = moment resultants of shell
N_x,N_y,N_{xy}, Q_x,Q_y = force resultants of shell
q = transverse distributed load
R = shell reference surface radius
r_i = distance between reference surface and the middle of the ith facing
u,v,w = displacements in the x, θ, and z directions, respectively
x,θ,z = shell coordinates
β_x,β_y = shell curvatures
$\sigma_{ix},\sigma_{i\theta},\sigma_{ix\theta}$ = stresses in the ith facing
$\sigma_{izx},\sigma_{iz\theta}$ = transverse shearing stresses in the ith core
$\nu_{x\theta}{}^i,\nu_{\theta x}{}^i$ = Poisson's ratios of the ith facing.

Introduction

MULTILAYER sandwich construction gained prominence in recent years for its use in space and atmospheric vehicles. The present analysis supplements earlier multilayer sandwich structure studies[1–3] in that it includes curved surfaces and assumes orthotropic properties in both facings and cores. The results of the investigation are presented in a form such that the shell elastic constants are of prime interest. The availability of the shell constants will allow existing solutions to be extended to include multilayer sandwich shells by a simple reidentification of the shell physical constants.

Figure 1 shows a section of a cylinder surface which consists of n facings and m cores. The facing and core thicknesses are denoted by $t_i(i = 1,2,3\ldots,n)$ and $h_i(i = 1,2,3\ldots,m)$, respectively. The facing and core materials are assumed to be orthotropic. Each layer (cores and facings) may possess different thicknesses and/or elastic properties.

Analysis

Following Donnell's approximation,[4] the equilibrium of moments and forces acting on a differential shell element will yield the equations

$$N_{x,x} + N_{\theta x,\theta}/R = 0 \tag{1a}$$

$$N_{x\theta,x} + N_{\theta,\theta}/R = 0 \tag{1b}$$

$$M_{x,x} + M_{\theta x,\theta}/R - Q_x = 0 \tag{1c}$$

$$M_{x\theta,x} + M_{\theta,\theta}/R - Q_\theta = 0 \tag{1d}$$

$$Q_{x,x} + Q_{\theta,\theta}/R - N_\theta/R + N_x w_{,xx} + N_\theta w_{,\theta\theta}/R^2 + 2N_{x\theta}w_{,x\theta}/R + q = 0 \tag{1e}$$

where comma denotes partial differentiation with respect to the following subscript.

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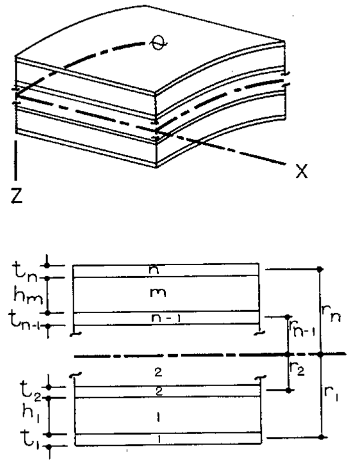

Fig. 1 Multilayer sandwich cylindrical shell element.

The stress resultants of the multilayered sandwich shell are defined as follows[1–3]:

$$[M_x;M_\theta;M_{x\theta}] = \sum_{i=1}^{n} t_i r_i[\sigma_{ix};\sigma_{i\theta};\sigma_{ix\theta}] \tag{2a,b,c}$$

$$[N_x;N_\theta;N_{x\theta}] = \sum_{i=1}^{n} t_i[\sigma_{ix};\sigma_{i\theta};\sigma_{ix\theta}] \tag{3a,b,c}$$

$$[Q_x;Q_\theta] = \sum_{i=1}^{m} h_i[\sigma_{izx};\sigma_{i\theta z}] \tag{4a,b}$$

Following the procedure as given in Refs. 1–3, the stresses in the ith layer are obtained and are given by

$$\sigma_{ix} = K_{11}{}^i(r_i\beta_{x,x} + u_{,x}) + K_{12}{}^i(r_i\beta_{\theta,\theta} + v_{,\theta} + w)/R \tag{5}$$

$$\sigma_{i\theta} = K_{12}(r_i\beta_{x,x} + u_{,x}) + K_{22}(r_i\beta_{\theta,\theta} + v_{,\theta} + w)/R \tag{6}$$

$$\sigma_{ix\theta} = K_{33}[r_i(\beta_{x,\theta} + \beta_{\theta,x})/R + v_{,x} + u_{,\theta}/R] \tag{7}$$

$$\sigma_{izx} = K_{44}{}^i(\beta_x + w_{,x}) \tag{8}$$

$$\sigma_{iz\theta} = K_{55}{}^i[\beta_\theta + (w_{,\theta} + v)/R] \tag{9}$$

where

$$\begin{aligned} K_{11}{}^i &= E_x{}^i/(1 - \nu_{x\theta}{}^i\nu_{\theta x}{}^i) \\ K_{12}{}^i &= \nu_{x\theta}{}^i E_x{}^i/(1 - \nu_{x\theta}\nu_{\theta x}) \\ K_{22}{}^i &= E_\theta{}^i/(1 - \nu_{x\theta}{}^i\nu_{\theta x}{}^i) \\ K_{33}{}^i &= G_{x\theta}{}^i \\ K_{44}{}^i &= G_{zx}{}^i \\ K_{55}{}^i &= G_{z\theta}{}^i \end{aligned} \tag{10}$$

Substitution of Eqs. (5–9) into Eqs. (1a–e) yields

$$M_x = D_{xx}(\beta_{x,x} + \nu_1\beta_{\theta,\theta}/R) + C_{xx}[u_{,x} + \nu_2(v_{,\theta} + w)/R] \tag{11}$$

$$M_\theta = D_{\theta\theta}(\beta_{\theta,\theta}/R + \nu_3\beta_{x,x}) + C_{\theta\theta}[(v_{,\theta} + w)/R + \nu_4 u_{,x}] \tag{12}$$

$$M_{x\theta} = D_{x\theta}(\beta_{\theta,x} + \beta_{x,\theta}/R) + C_{x\theta}(v_{,x} + u_{,\theta}/R) \tag{13}$$

$$N_x = B_{xx}[u_{,x} + \nu_5(v_{,\theta} + w)/R] + C_{xx}(\beta_{x,x} + \nu_2\beta_{\theta,\theta}/R) \tag{14}$$

$$N_\theta = B_{\theta\theta}[(v_{,\theta} + w)/R + \nu_6 u_{,x}] + C_{\theta\theta}(\beta_{\theta,\theta}/R + \nu_4\beta_{x,x}) \tag{15}$$

$$N_{x\theta} = B_{x\theta}(v_{,x} + u_{,\theta}/R) + C_{x\theta}(\beta_{\theta,x} + \beta_{x,\theta}/R) \tag{16}$$

$$Q_x = D_{zx}(\beta_x + w_{,x}) \tag{17}$$

$$Q_\theta = D_{\theta z}[\beta_\theta + (v + w_{,\theta})/R] \tag{18}$$

where the elastic constants, B_{ij}, C_{ij}, D_{ij} and the modified

Poisson's ratios ν_i appearing in Eqs. (11–18) are given by

$$[D_{xx};D_{\theta\theta};D_{x\theta}] = \sum_{i=1}^{n} t_i r_i^2 [K_{11}^i;K_{22}^i;K_{33}^i] \quad (19)$$

$$[C_{xx};C_{\theta\theta};C_{x\theta}] = \sum_{i=1}^{n} t_i r_i [K_{11}^i;K_{22}^i;K_{33}^i] \quad (20)$$

$$[B_{xx};B_{\theta\theta};B_{x\theta}] = \sum_{i=1}^{n} t_i [K_{11}^i;K_{22}^i;K_{33}^i] \quad (21)$$

$$[D_{zx};D_{\theta z}] = \sum_{i=1}^{m} h_i [K_{44}^i;K_{55}^i] \quad (22)$$

$$\nu_1 = \frac{1}{D_{xx}} \sum_{i=1}^{n} t_i r_i^2 K_{12}^i; \quad \nu_2 = \frac{1}{C_{xx}} \sum_{i=1}^{n} t_i r_i K_{12}^i$$

$$\nu_3 = \frac{1}{D_{\theta\theta}} \sum_{i=1}^{n} t_i r_i^2 K_{22}^i; \quad \nu_4 = \frac{1}{C_{\theta\theta}} \sum_{i=1}^{n} t_i r_i K_{12}^i \quad (23)$$

$$\nu_5 = \frac{1}{B_{xx}} \sum_{i=1}^{n} t_i K_{12}^i; \quad \nu_6 = \frac{1}{B_{\theta\theta}} \sum_{i=1}^{n} t_i K_{12}^i$$

Equations (19, 21, and 22) give the multilayer sandwich shell stiffness in bending, extension, and shear, respectively. Equation (20) defines the coupling stiffnesses of the shell. Equation (22) gives the transverse shearing stiffnesses; and Eq. (23) defines the modified Poisson's ratio of the shell.

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Boundary-Layer Induced Pressures on a Flat Plate in Unsteady Hypersonic Flight

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Introduction

In this Note a procedure is developed for the calculation of hypersonic viscous interaction pressures for unsteady flows, valid for weak interactions. This procedure is based on an extension of the idea underlying the tangent wedge approximation which is generally accepted for the calculation of interaction pressures in steady flows. Examples are then worked out, using this method, to illustrate the unsteady features of the interaction pressures. The simple, yet nontrivial, flow configuration of a semi-infinite flat plate moving in its own plane with various situations of motion unsteadiness is chosen to demonstrate the idea, and use is made of the available solutions for such unsteady boundary-layer flows without interactions. Results are reported for the first approximation of weak interaction pressures for the unsteady situations considered. For this class of flow geometry it is obvious that the effects of flow unsteadiness on the aerodynamic forces do not appear in an inviscid analysis. It is believed that the present study is useful for the determination of various aerodynamic forces on hypersonic vehicles in unsteady flight, and that the results reported here are new. Many details of the analysis and calculations are omitted in this Note but appear in Ref. 1.

Analysis

Let the displacement surface of an unsteady boundary layer with no interaction be denoted by

$$F(x,y,t) \equiv y - \Delta(x,t) = 0 \quad (1)$$

where the rectangular Cartesian coordinate system (x,y) is fixed with respect to the plate, with its origin at the leading edge of the plate. The plate is moving with a continuously varying (otherwise arbitrary) velocity, $U(t)$, in the negative x direction. Then the first approximation of the weak interaction pressure on such a plate is just the pressure on the two-dimensional unsteady surface $y = \Delta(x,t)$, which moves in the negative x direction at a hypersonic Mach number, $M(t) = U(t)/a_\infty \gg 1$, a_∞ being the ambient sound speed.

To proceed, we shall first indicate how the "weak" form of the tangent wedge approximation used in Ref. 2 to calculate weak interaction pressures for steady flows can be extended and employed for the present unsteady flows.

We note that the tangent wedge formula in its normal form corresponds to the leading approximation to the exact solution of supersonic wedge flow in the limit of $\theta_b \to 0$, $M_\infty \to \infty$ with $K_b \equiv M_\infty\theta_b$ fixed, where θ_b is to be taken as the local body inclination angle with the freestream and M_∞ is the freestream Mach number, U/a_∞. The surface pressure thus obtained is the same as that on a one-dimensional piston pushing into still air with a velocity equal to $U\theta_b$, the approximate normal velocity of the surface at the local point under consideration. The weak form of the tangent wedge approximation, which corresponds to the limit $\theta_b \to 0$, $M_\infty \to \infty$ with $K_b \to 0$, is thus equivalent to the further approximation of low piston Mach number, i.e., $U\theta_b/a_\infty \ll 1$, in the one-dimensional piston problem.

By relating the results of tangent wedge approximation to those of the one-dimensional piston problem as in the aforementioned, the idea of the tangent wedge approximation can be easily extended to unsteady flows by simply regarding the surface element as a piston moving with a time dependent speed equal to the normal velocity of the unsteadily moving surface at the particular point concerned. In particular, the first approximation of the surface pressure for unsteady, weak interactions can thus be taken to be the acoustic pressure on a piston moving at a variable, but low, speed.

It may be worth mentioning here that, by following this reasoning, it is possible to extend the "strong" form of the tangent wedge approximation used in Ref. 2 to calculate the steady strong interaction pressures to handle the unsteady strong interaction problems. Here we have the situation of $K_b \gg 1$, which corresponds to the case of a piston pushing with a very high (time dependent) Mach number into air at rest. Then the so called "snowplow theory," for example, might be one of the possibilities for providing a relationship

Table 1 Values of k, k_0, and k_1

	Insulated	Isothermal
k	0.56	0.14
k_0	−2.83	−0.23
k_1	5.10	0.33

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